Step input response application on control of distillate and reflux ratio of a distillation column

C. P. Ukpaka and Okaniba, O. Bestman

1Department of Chemical/Petrochemical Engineering, Rivers State University of Science and Technology, Nkpolu, P.M.B. 5080, Port Harcourt, Nigeria
2Department of Chemical/Petroleum Engineering, Niger Delta University Wilberforce Island, Bayelsa State

*Corresponding author E-mail: chukwuemeka24@yahoo.com

INTRODUCTION

In this research work, the following components are useful for the research work which includes; distillation column, mathematical model, distillate product, reflux ratio, as well as the application of step input responses. Distillation column is often considered as the most significant unit and most commonly used in separation processes in terms of chemical application as well as separating feed streams and for the purpose of purification of final and intermediate product from the streams and for the purpose of purification of final and intermediate product from the streams.

The many objectives of a distillation unit is the ability to separate a mixture of components with the application of heat into different products of more than two or more, with different chemical compositions. The distillation column is made up of the following component’s, entering of the feed in to the distillation column which contains the trays, the mechanisms of distillate components and the bottom components. For the distillate components will have the condenser, storage tank, reflux and distillate product separator with the reflux flow containing a control valve. In the bottom distillate it contains the reboiler, heater with a control valve as well as recovery of the bottom product as the residual product which may be useful for other application.
The separation takes place in a vertical column where heat is introduced to a reboiler at the bottom and removed from condenser at the top. A stream of vapor produced in the reboiler rises through the column and is forced into contact with a liquid stream from the condenser flowing downwards in the column. The volatile (light) components are enriched in the vapors phase whereas the less volatile (heavy) components are enriched in the liquid phase (James, 2008).

**The distillate products**: The distillate products are achieved as stated above. The mechanism to achieve the distillate products is known as the application of fractional distillation. The mechanism involves separation of the distillate product based on the boiling points; reflux ratio, control, and control responses (Ghydaa Majeed Jaid 2009; Ukpaka et. al 2008, 2010).

**Reflux Ratio**: As stated above, as the feed is heated through the reboiler, the more volatile component evaporates up the column, and the condensed in the condenser, from the condenser the condensed products enters into the reflux storage tanks, from the reflux storages tank some of the product is recycled back into the fractionators. The recycled product is known as (Refux) while the remaining products come out as distillate product (Ukpaka and Nnadi, 2008, Ukpaka,Ukpaka, 2011).

Therefore, reflux ratio is the ratio of reflux flow to that of distillate flow (Warren, et. al, 1993).

**Control of a distillation unit**: The control of a distillation unit is a very important since it have a significant impact on the improvement of the product from the unit, also proper control of the unit will help to improve the efficiency of the distillation unit by reducing the energy consumed, there will be protecting of the environmental resources (Ukpaka and Farrow, 2007 & Ukpaka and Ndor, 2013).

Distillation control is a challenging problem due to the following factors: process non-linearity, substantial coupling of manipulated variables, No stationary behavior and severe disturbances

From the research works done by other researchers on the control of a distillation unit it is found that control has the major impact on the quality of product, energy usage and plant through put on those industries, also effective control of a distillation unit is the best way to reduce the operation cost of an existing unit since distillation processes requires large amount of energy (Ghydaa, 2012).

**Control of response**: There are various types of response; some of these are step input response: There are functions those changes instantaneously from one level to another. If the step size is equal to one (1), then it is called a unit step function, ramp inputs: These are functions that changes linearly with time, impulse: This is a Dirac delta function on infinitely high pulse whose width is zero and area is unity e.g. infection of a finite amount of energy into process in an infinitesimal length of time, sinusoidal input: These are input that vary periodically, pulse: Pulse is a function that begins and ends at the same level, it is the sum of one positive step function made at time zero and one negative step function made DL minutes later BL is a delay or dead time, and exponential input (Ogoni and Ukpaka 2004).

The aim of the study is to develop a mathematical model that will predict, control and monitor the dynamic response of a distillation unit under extreme transient conditions, and to demonstrate the usefulness of the model for various control response of the column, the calculated result will be compared with those obtained experimentally for a multi component system. The objectives of this research work are as follows: Product quality control: Both the products from the top and bottom have to be controlled to obtain the specified percentage of compositions at the top and bottom product. Material balance control: The column control system must cause the average sum of the distillate and tail streams to be exactly equal to the average feed rate; also, the resulting adjustments in process flow must be smooth and gradual to avoid upsetting either the column or downstream process equipment fed by the column.

**Constraints**: For safe satisfactory operation of the column, certain constraints must be observed for example, the column should not flood and Column pressure drop should be high enough to maintain effective. This project is concerned with control implemented using various control strategies. The work will look into the control methods and control responses for step input response and there results, the dynamic mode for a distillation unit is to be studied by introducing step changing in reflux rate also measuring the distillate and reflux concentration, studying of the interaction of variables and selecting the best control parameters by carrying a tuning procedures using different control equipment.

Distillation is an important process in chemical engineering, hence, need to be improved to increase efficiency, to enhance quality product, if this is not done, then, product of low quality may be achieved even though the need arise to produce product that have high quality than what is been produced in the past, this is because in separation of a mixture of component with slightly different volatility may overlap thereby reducing the quality of the product, therefore, the distillation unit have to be controlled and monitor properly. This study unlike others will enable us develop a mathematical model to control and monitor the distillation unit, with the aim to get quality of the product.

Also Joao (2009) studied modeling and intelligent control of a distillation column. In his work, he studied two main types of modeling paradigms were addressed, the linear state space models and non linear models where addressed, these
models were estimated directly from experimental data and simulated data.

Ogoni and Ukpaka, (2004) studied transient response of a distillation column, he studied the dynamics of a distillation column, input disturbances such as: step function, impulses function, logarithmic functions and trigonometric function. Satish, (2000), in his research, he studied distillation variability prediction where he used novel techniques to predict product variability for distillation columns. The method uses industrial disturbance data and applies signal processing techniques to extract its amplitude and frequency information. This information is combined with the closed-loop Bode plot for the same disturbance as a function of frequency to predict closed-loop product variability for the column. The closed-loop Bode plot is obtained using a linear dynamic model of the process. The approach is demonstrated using a binary distillation column, a C3 splitter and a multicomponent distillation column, a depropanizer. Four different designs of both columns were considered. A thorough study of the approach is carried out to verify the accuracy and the shortcomings of the approach. The potential of the approach as a quantitative tool for configuration selection was also explored. For this purpose, nine different distillation configurations were analyzed which indicated that this approach can be successfully used for distillation configuration selection.

**MATERIALS AND METHODS**

The mathematical model to be developed in this section will include the mass and energy balance equations of the feed stream the trays, the reboiler the condenser, and the reflux tank that is responsible for the distillate and reflux that goes back into the fractionators, the feed to be considered for this job is the Nigerian crude oil.

**Development of mathematical model**

**Assumptions used in this modeling**

1. The feed stream considered as stated above is the Nigerian crude which contains more than two components.
2. The column preserve is assumed to be at steady state for each thing but changes linearly throughout the unit.
3. The vapor hold up is negligible.
4. The unit has a singular period input and two product streams, which are the over head or top product and the bottom or heavy product.
5. At each tray and the reboiler the vapor phase is assumed to be in equilibrium with the liquid phase
6. The equation of state used to represent the vapor liquid equilibrium is that of soave-peddlish-among.
7. Enthalpies are estimated using ideal enthalpy data and enthalpy departure functions.
8. The reboiler is treated as an ideal stage
9. Murphee tray efficiency are used for the modeling of the trays because they are assume d to be non ide al.
10. Hydraulic time constant approach is used to treat the dynamics of the liquid tray.
11. Heat transfer dynamics of the condenser are not considered
12. The heat transfer dynamics of reboiler are treated using first-order lag.
13. The column has total condenser and partial reboiler
14. Flow rate model with first order lags to represent value dynamics
15. Tray temperatures are used to infer overhead and bottom compositions for control.
16. The heat transfer dynamics of the tray temperatures measurements are modeled using first order stage
17. Sensible heat change at each tray of the column is assumed to be small enough to be rejected.

**Vapour/Liquid equilibrium**

The figure 1 above has a single feed stream and double product stream which comprises of the top and bottom product. At each tray of the fractionators the liquid composition is in equilibrium state with the vapor composition.

In this distillation unit we have the following component,

i. The feed or inlet
ii. The trays
iii. The reboiler
iv. The condenser
v. The reflux drum

For a multi-component mixture such as crude oil, the vapor liquid equilibrium (VLE) between these components can be represented by:

\[
\hat{f}_v = \hat{f}_l, \text{ where } \hat{f}_v \text{ is the fugacity of the vapour component and } \hat{f}_l \text{ is the fugacity of the liquid component.}
\]

Expressing the fugacity in terms of fugacity coefficient \( y \phi^v \) (1)

Where \( \phi^v \) and \( \phi^l \) and fugacity coefficient of vapour and liquid fugacity are estimated using Redlich kwong equation of state.

Using Redlich Kwong equation of state fugacity coefficient can be represented as \( Y_{e} = K_{ij} X_{ij} \) (2)
Where $K$ = vaporization equilibrium ratio and is defined as

\[ K_{ij} = \frac{\phi_{ij}}{\phi_{ij}} \]

Also for a multicomponent system the relative volatility is defined as:

\[ \alpha_{nj} = \frac{K_{nj}}{K_{nj}} \quad \text{(3)} \]

Where $\alpha_{nj}$ = the relative volatility of component n and j

The above vapor liquid equilibrium equations above (VLE) are solved on each tray. For us to have a linear dynamic model, these equations have to be linearized at each tray around its steady state operating conditions.

Using numerical techniques for the linearization of the equations above, we have

\[ \bar{y}_j = \left[ (K_{ij}) \bar{x}_y + \sum \left( x_{iy} \frac{\partial y_{ij}}{\partial x_{ij}} \right) \bar{x}_{ik} + \sum \left( x_{ji} \frac{\partial y_{ij}}{\partial y_{ik}} \right) \bar{y}_{ik} + \left( x_{ij} \frac{\partial y_{ij}}{\partial T} \right) T \right] \quad \text{(4)} \]

This equation is solved using numerical techniques. The equations are linear but solving these equations for each tray needs a long amount of computational period, to reduce the amount of computational time an inside-out algorithm is used.

The modified version of the $K_B$ method is used for a multicomponent distillation unit and it is given as:

\[ \ln (K_{Bi}) = A_j - \frac{B_i}{T_j} \quad \text{(5)} \]

Using the basic rules of vapor liquid equilibrium for a given tray n and components it can be shown that $K_B$ is given by

\[ \ln (K_B) = \sum_i 0 \gamma_{ij} \ln (K_{ij}) \quad \text{(6)} \]

To calculate the parameters $A$ and $B$ the linear vapor liquid equilibrium equations are solved at two temperatures close to the tray temperature $T$ and $T + \Delta T$, $\Delta T$ being small.

Using equation (5) and (6) the parameters are calculated as

\[ B_i = \frac{\sum \gamma_{ij} \ln (K_{ij}) + \Delta T}{\sum \gamma_{ij} \ln (K_{ij}T)} \quad \text{(7)} \]

\[ A_i = \sum \gamma_{ij} \ln (K_{ij}) + \frac{B_i}{T} \quad \text{(8)} \]

**Linear modeling**

Using the input-out method as shown in the Fig. 1b and Fig. 1a is showing simple distillation column.

Mass balance equation for stage n is given as

\[ F_n + V_{n+1} + L_{n-1} = V_n + L_n \quad \text{(9)} \]

The component balance for stage n is given as

\[ F_n Z_n + V_{n+1} Y_{n+1} + L_{n-1} X_{n-1} = V_n Y_n + L_n X_n \quad \text{(10)} \]

**Linear modeling for the interior tray**
Using the input-output method, for the \( n \)th tray the mass balance equation is:

\[
\left( \text{Rate of mass into the system} \right) - \left( \text{Rate of mass out of the system} \right) = \left( \text{Rate of accumulation of mass} \right)
\]

Mathematically

\[
L_{n+1} + V_{n-1}L_n - V_n = \frac{dm_n}{dt} \quad \text{(11)}
\]

Equation (11) is the material balance equation of the \( n \)th tray at unsteady state. For us to develop the model for the trays, we have to make the system a steady state system therefore

\[
\frac{dm_n}{dt} = 0 \quad \text{(12)}
\]

Then equation (11) becomes,

\[
L_{n+1} + V_{n-1}L_n - V_n = 0 \quad \text{(13)}
\]

For the energy balance equation

\[
\frac{dh_n}{dt} = h_{n+1}L_{n+1} - h_{n}L_n + V_{n-1}H_n - V_nH_n \quad \text{(14)}
\]

Becomes

\[
\frac{dh_n}{dt} + \frac{h_n(dm_n)}{dt} = h_{n+1}L_{n+1} - h_{n}L_n + V_{n-1}H_n - V_nH_n \quad \text{(15)}
\]

\( \frac{dh_n}{dt} \) is the sensible heat change on each tray. This term can be negligible for a multicomponent distillation column. Then equation (15) becomes

\[
\frac{dm_n}{dt} = h_{n+1}L_{n+1} - h_{n}L_n + V_{n-1}H_n - V_nH_n \quad \text{(16)}
\]

Substituting equation (13) into (9) we have,

\[
(h_{n+1} + V_{n-1} - L_n - V_n) = h_{n+1} - h_nL_n + V_{n-1}H_n - V_nH_n \quad \text{(17)}
\]

Simplify equation (17) we have

\[
V_n = \frac{L_{n+1}(h_{n+1} - h_n) + V_{n-1}(h_{n-1} - h_n)}{h_n - h_n} \quad \text{(18)}
\]

The liquid and vapor enthalpies are obtained from ideal enthalpy data using departure function Satish, (2000), these departure functions are estimated using (SRK) equation of state. Satish, (2000) thus the enthalpy equation for each component in liquid and vapor are given as

\[
\dot{h}_{n,1} = \dot{h}_{n,1} + Dh_{n,1} \quad \text{(19)}
\]

\[
\dot{H}_{n,1} = \dot{H}_{n,1} + DH_{n,1} \quad \text{(20)}
\]

Where, \( \dot{h}_{n,1} \) = The ideal liquid enthalpy component 1, \( Dh_{n,1} \) = This represent the departure of liquid enthalpy of component 1 at tray n from an ideal solution. This statement is applicable for vapor enthalpy.

For a multicomponent liquid and vapor stream are assumed to be ideal solution, therefore the liquid and vapor stream enthalpies are given respectively as

\[
h_n = \sum \dot{h}_{n,1} \quad \text{(21)}
\]

\[
H_n = \sum \dot{H}_{n,1} \quad \text{(22)}
\]

To obtain a linear mode for the depropanizer the non-linear equations at each tray are linearized around the steady state operating conditions on each tray, the linearized equations are:

For the mass balance equation, the linear model is

\[
\frac{dM_n}{dt} = L_{n+1} + V_{n-1} - L_n - V_n \quad \text{(23)}
\]

For the energy balance linear equation \( H \) is given as

\[
\frac{dM_n}{dt} = \bar{V}_n - \bar{L}_R - \bar{D} \quad \text{(24)}
\]

The component material balance is

\[
\frac{d(x_{ACC}M_{ACC})}{dt} = Y_nV_n - (L_R + D)X_{ACC} \quad \text{(25)}
\]

The reflux drum is considered to be in an ideal stage, also it is assumed that the cooling water is not considered as part of the distillation unit, thereafter the energy balance equation is not put into consideration.

The linear model for the reflux drum is given by

\[
\frac{dM_{ACC}}{dt} = \bar{V}_n + \bar{L}_R + \bar{D} = 0 \quad \text{(26)}
\]

\[
\frac{dM_{ACC}}{dt} = \bar{V}_n + \bar{L}_R + \bar{D} \quad \text{(27)}
\]

Also the component balance for the linear model is given as

\[
\frac{d(x_{ACC}M_{ACC})}{dt} = (Y_n)V_n - (V_n)X_{ACC} \quad \text{(28)}
\]

Dividing through equation (28) by \( M_{ACC} \) we have

\[
\frac{d(x_{ACC})}{dt} = \left( \frac{V_n}{M_{ACC}} \right) \frac{\bar{V}_n}{V_n} - \left( \frac{V_n}{M_{ACC}} \right)X_{ACC} \quad \text{(29)}
\]

\[
\frac{dX_{ACC}}{dt} = \left( \frac{V_n}{M_{ACC}} \right)(\bar{V}_n - X_{ACC}) \quad \text{(30)}
\]
Modeling of the reboiler

For the reboiler the energy and material balance equation are necessary, they are given as:

\[ \frac{dM_{REB}}{dt} = L_i - V_o - B - \cdots \quad (31) \]

The component material balance, is given as,

\[ \frac{d(X_{REB}M_{REB})}{dt} = X_i L_i - Y_{REB} V_0 - X_{REB} B - \cdots \quad (32) \]

For energy balance,

\[ V_o = \frac{Q + \dot{H}_{L} (\bar{h}_{L} - h_{REB}) - M_{REB} \frac{dM_{REB}}{dt}}{M_{REB} - h_{REB}} \]

Also assuming that reboiler is in ideal state, the resulting modeled equation is

\[ \frac{d\bar{M}}{dt} \cdot L_i + \bar{V}_o + \bar{B} = 0 \quad - \quad (33) \]

\[ \bar{L}_1 - \bar{V}_o - \bar{B} = \frac{d\bar{M}}{dt} \quad - \quad (34) \]

\[ \frac{d(X_{REB}\bar{M}_{REB})}{dt} = (X_i - X_{REB}) \frac{L_i}{M_{REB}} + (L1) \bar{X} - r(Y_{REB} - X_{REB}) - (L1 - VO) \bar{X}_{REB} (V_o) \bar{Y}_{REB} \quad (35) \]

Dividing through equation (35) by \( \bar{M}_{REB} \) we have

\[ \frac{d(X_{REB})}{dt} = \frac{x - x_{REB}}{M_{REB}} \frac{L_i}{M_{REB}} \frac{X_i}{M_{REB}} - \frac{Y_{REB} - X_{REB}}{M_{REB}} \bar{X}_{REB} - \frac{V_o}{M_{REB}} \bar{Y}_{REB} \quad (36) \]

### Solutions to modeled equations

In this section we are going to use various mathematical tools to solve the modeled equations.

Solution to modeled equation for the interior tray we have,

\[ \frac{dM_n}{dt} = L_{n+1} + V_{o+1} - L_{n} - V_{n} \quad (36a) \]

Assuming a constant molar overflow, where the mixture has a constant heat of vaporization and that sensible heat of mixing effect are negligible (Hery, 1992).

\[ L_{n+1} = L_n = L \quad - \quad (37) \]

Where \( L \)= Liquid flow rate into the distillation column

Also

\[ V_{n+1} = V_n = V \quad - \quad (38) \]

Substituting equation (41) and (24) into equation (11) we have

Each tray is assumed to be a CSTR with a hydraulic time constant \( t_{tray} \), hence equation (36a) becomes

\[ \frac{dL_n}{dt} = \frac{L_{n+1} + V_{o+1} - L_n - V_n}{t_{tray}} \quad (39) \]

\[ t \frac{dL_n}{dt} = (L_{n+1} + V_{o+1}) - (L_n + V_n) \quad (40) \]

Where, \( L_n \) = Liquid hold up on stage \( n \), \( t \) = Time, \( L_n \), \( V_n \) = Input liquid and vapour flow rates, \( L_{n+1}, V_{o+1} \) = Output liquid and vapour flow rates

From equation (40) we have

\[ \frac{t dL_n}{dt} = L_{n+1} + V_{o+1} - L_n - V_n \quad - \quad - \quad (41) \]

\[ \frac{t dL_n}{dt} = (L_{n+1} - L_n) + (V_{o+1} - V_n) \quad - \quad (42) \]

Substituting equation (40) and (41) into equation (42), we have,

\[ \frac{t dL_n}{dt} = L_n + V_n \quad - \quad (43) \]

Using laplace transform to solve equation (43) we have

\[ \frac{dL_n}{dt} = SL_n(s) - L_{n(o)} \quad - \quad (44) \]

\[ L_n = L_{n(s)} \quad - \quad (45) \]

\[ V_n = V_{n(s)} \quad - \quad (46) \]

Substituting equation (44, 45 and 46) into equation (43) we have

\[ t (SL_n(s) - L_{n(o)}) = L_{n(s)} + V_{n(s)} \quad - \quad (47) \]

\[ t s L_{n(s)} = L_{n(o)} + V_{n(s)} \quad - \quad (48) \]

\[ t s L_{n(s)} = L_{n(o)} + V_{n(s)} \quad - \quad (49) \]

\[ L_{n(o)} (s-I) = V_{n(s)} \quad - \quad (50) \]

\[ L_{n(s)} = \frac{KV_{n(s)}}{s-I} \quad - \quad (51) \]

Equation (53) is the laplace transform equation

\[ L_{n(s)} = \frac{KV_{n(s)}}{s-I} \quad - \quad (51) \]

Where, \( L_{n(o)} \) = Output response, \( V_{n(s)} \) = Input response, \( K = \) Gain = 1

### Solution to the modeled equation for the reflux drum

From equation (10) we have

\[ \frac{dM_{acc}}{dt} = \dot{V}_n \cdot L_R - D \quad (52) \]

Where, \( V_n \) = Vapour flowrate, \( L_R \) = Liquid flowrate, \( D \) = Distillat flow rate
Dividing through equation (53) by D, we have

\[ \frac{1}{D} \frac{dM_{\text{acc}}}{dt} = \frac{\bar{V}_n}{D} - \frac{L_R}{D} - 1 \]  \hspace{1cm} (54) \]

Substituting equation (56) into equation (55) we have

\[ \frac{1}{D} \frac{dM_{\text{acc}}}{dt} = \frac{\bar{V}_n}{D} - (R + 1) \]  \hspace{1cm} (58)
Let \( t = \frac{1}{D} \). \hfill (59)

Substituting equation (59) into (58), we have
\[
\frac{dM_{acc}}{dt} = \frac{V_n}{D} - (R + 1)
\]

If \( \frac{V_n}{D} \) is normalized to be a function of mass dependent
\[
\frac{V_n}{D} = \frac{m^3}{S} \times \frac{kgS}{m^3}
\] \hfill (60)

Where equation 60 represent density per unit time \( \frac{S}{t} \)

Therefore we have,
\[
\frac{dM_{acc}}{dt} = \frac{V_n}{D} + \frac{MD}{V_n} - (R + 1) - \hfill (61)
\]

\[
\frac{dM_{acc}}{dt} = M_{acc} - (R + 1) - \hfill (61a)
\]

let \( y = M_{acc} \) and \( R + 1 = Y \)

Therefore
\[
\left\{ \begin{array}{l}
t \left( \frac{dy}{dt} \right) y - u = - \\
y = y(s) - \\
u = u(s)
\end{array} \right. \hfill (61b)
\]

Substituting equation (62) into equation (61b) we have
\[
t(SY(s) - Y(s)) = Y(s) - U(s) - \hfill (62)
\]

\[
tSY(s) = Y(s) - Y(s)
\]

\[
tSY(s) - Y(s) = - U(s)
\]

\[
Y(s) - tSY(s) = U(s)
\]

\[
Y(s)(1-tS) = U(s)
\]

\[
Y(s) = \frac{RI(s)}{S-\nu}
\]

Solution to modeled equation for the reboiler the material balance equation for the reboiler is
\[
\frac{dM_{reb}}{dt} = L_n - \nu - B \hfill (64)
\]

Substituting equation (65) into equation (64), we have
\[
\frac{dM_{reb}}{dt} = L_n - L - D - B - \hfill (66)
\]

Assuming a constant molar overflow
\[
L_n + L
\]

Rearranging equation (66), we have
\[
\frac{dM_{reb}}{dt} = (L_n - L) - D - B \hfill (67)
\]

But \( \text{Relux} = R = \frac{L}{D} \)

Therefore \( L = RD \) \hfill (68)

Substituting equation (67) into (68) we have
\[
\frac{dM_{reb}}{dt} = RD - D - B \hfill (69)
\]

Dividing through equation (69) by \( D \) we have
\[
\frac{1}{D} \frac{dM_{reb}}{dt} = (R - 1) - \frac{B}{D} \hfill (70)
\]

Let
\[
\frac{1}{D} = \tau_0
\]

Equation (70) becomes
\[
\tau_0 \frac{dM_{reb}}{dt} = (R - 1) - \frac{B}{D}
\]

If \( \frac{B}{D} \) is normalized to be a function of mass dependent
\[
\frac{B}{D} = \frac{m^3}{S} \times \frac{kgS}{m^3} \hfill (71)
\]

\[
\tau_0 \frac{dM_{reb}}{dt} = (R - 1) - \frac{M_{reb}}{B}
\]

Let
\[
M_{reb} = Y \text{ and } (R - 1) = U
\]

\[
\tau_0 \frac{dy}{dt} = U \cdot Y \hfill (72)
\]

Applying laplace transform to equation (72), we have
\[
\frac{dy}{dt} = SY(s) - Y(s) \hfill (73)
\]

\[
U = U(s) \hfill (73)
\]

\[
Y = Y(s) \hfill (73)
\]
Substituting equation (73) into equation (72), we have

\[ Y(s)(SY(s) - Y(o)) = U(s) - Y(s) \]

Using boundary condition

\[ Y(o) = 0 \]

\[ Y(s)(YSY(s) + Y(s)) = U(s) \]

\[ Y(s)(TS + 1) = U(s) \]

\[ Y(s) = \frac{KU(s)}{TS + 1} \] (73)

**Control Response**

Some of the responses to consider at this point are

1. **Step input response**: These are function that changes instantly from one level to the other. When the step size is equal to 1 then it is called a unit step function

\[ U(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t \leq 0 \end{cases} \]

\[ U(s) = \frac{A}{S} \]

(for forcing functions)

**Applying different control response to the interior tray**

1. **Step input response**

From equation (53) we have

\[ Ln(s) = \frac{KVP(s)}{TS - 1} \] (74)

Also, for step input response

\[ U(s) = \frac{A}{S} \] (75)

From equation (74), Let

\[ U(s) = Vn(s) \]

and \[ Y(s) = Ln(s) \]

Therefore

\[ Y(s) = \frac{KU(s)}{TS - 1} \] (76)

Substituting question (75) into equation (76) we have

\[ U(s) = \frac{KA}{S(TS - 1)} \] (77)

Applying laplace partial function to equation (77) we have

\[ \frac{KA}{S(TS - 1)} = \frac{B}{S} + \frac{C}{TS - 1} \] (78)

Solving the partial fraction

\[ KA = B(TS - 1) + CS \]

Equating coefficients

\[ KA = BrS - B + CS \]

From equation (79) we have

\[ B = -KA \]

Substituting the B into equation (80) we have

\[ 0 = \tau KA + C \]

Therefore

\[ C = \tau KA \]

Substituting the values of B and C into equation (78) we have

\[ \frac{KA}{S(TS - 1)} = \frac{KA}{S} + \frac{\tau KA}{TS - 1} \] (81)

\[ Y(s) = \frac{A}{S(TS - 1)} \]

\[ Y(s) = \frac{\tau KA}{S} \]

\[ Y(s) = \frac{KA}{S(TS - 1)} \]

\[ Y(s) = KA(e^{-\tauT} - 1) \] (82)

Where, \( K = \text{gain}, A = \text{Amplitudes} \)

Applying Different Control Response to the Reflux Drum

1. **Step input Response**

For step input response,

\[ U(s) = \frac{A}{S} \]

But from equation (73) we have

\[ Y(s) = \frac{KU(s)}{TS + 1} \]

Therefore

\[ Y(s) = \frac{KA}{S(TS + 1)} \] (83)

Applying partial fraction to equation (83) we have

\[ \frac{KA}{S(TS + 1)} = \frac{B}{S} + \frac{B}{TS + 1} \] (84)
Table 2: Relationship among area, time and residence time for the response investigated

<table>
<thead>
<tr>
<th>Area (m²)</th>
<th>Time (sec)</th>
<th>Residence time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td></td>
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<tr>
<td>5</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td></td>
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<tr>
<td>7</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

For the reboiler, from equation (73) we have

\[ Y(s) = \frac{KA}{s^2 + 1} \]

Considering the following control response

1. Step input response:

For step input response

\[ U(s) = \frac{A}{s} \]

Therefore, the equation for the reboiler becomes

\[ Y(s) = \frac{KA}{s(s+1)} \]

Applying partial fraction to equation (91)

\[ \frac{KA}{s(s+1)} = B + \frac{C}{s+1} \]

Equating coefficient from equation (93) we have

\[ KA = B(\tau T + 1) + CS \]

\[ KA = BrS + B + CB \]

Solving equation (94) and (93) simultaneously, we have;

\[ B = KA \]

\[ C = KAr \]

Substituting equation (96) into (92) we have

\[ \frac{KA}{s(s+1)} = Y(s) = \frac{KAT}{s+1} \]

Taking inverse laplace, transform from equation (98) we have

\[ Y(s) = \frac{KA}{s} - \frac{KAr}{s+1} \]

The results obtained from the modeled equation. The results obtained from this modeled equation are gotten by testing the models with experimental data. The first part of this chapter shows the results of step input for the feed tray

**Step input for the feed tray:** Figure 1a,b,c, and d shows the result tested models with different input parameters,
figure(1a) represent when the area is two meter square (2m$^2$), figure(1b) represent when the area is four meter square (4m$^2$), figure(1c) represent when the area is six meter square (6m$^2$) and figure(1d) represent when the area is eight meter square (8m$^2$).

From table 2 the graph below is plotted. From the graph demonstrated in Figure 3a,b,c,d, it is seen that for step input response the output $y(t)$ increase with increase in time, but at time equal to 6.5s the output tend to drop and at time above 6.5s the output rate is increased. Also for equal time range (0 to 10s) and varying area we have different value of output rate, we can also see that increase in area also result to increase in output response from the figure above. The behavior is due to the properties of the crude, some of these properties are the viscosity of the crude oil, and the sulphur content of the crud, the presence of salt in the crud is also a
major contributing factor for the behavior of the output response as presented in figure 3a,b,c,d.

**Testing of modelled equation for the reflux drum**

The figures below shows the result for tested models with different input parameters with varying area as follows, when the area is two meter square (2m$^2$), f when the area is four meter square (4m$^2$) when the area is six meter square (6m$^2$) and when the area is eight meter square (8m$^2$). The figures below presents the graphs for step input response, ramp input, impulse input, pulse input and exponential input for the reflux drum as presented in figure 4a,b,c,d.

**Plot for step input for the reflux drum:** the plot below represent a plot of output against time using step input function.

From the figure 4a,b,c,d, it is shown that for step input response the output $y(t)$ increase with increase in time, but at time equal to 8.3s the output tend to drop and at time above 8.3s the output rate is increased. Also for equal time range (0 to 10s) and varying area we have different value of output rate, we can also see that increase in area also result to increase in output response from the figure above. The behavior is due to the properties of the crude, some of this properties are the viscosity of the crude oil, the sulphur content of the crude, the presence of salt in the crude is also a major contributing factor for the behavior of the output response.

**Testing of modelled equation for the reboiller**

The figures below shows the result for tested models with different input parameters with varying area as follows, when the area is two meter square (2m$^2$), f when the area is four meter square (4m$^2$) when the area is six meter square (6m$^2$) and when the area is eight meter square (8m$^2$). The figure 4 presents the graphs for step input response, ramp input, impulse input, pulse input and exponential input for the reboiller.

**Plot for Step Input for the Reboiller:** The plot below represent a plot of output against time using step input function.

From the graph demonstrated in Figure 5a,b,c,d it is seen that for step input response the output $y(t)$ increase with increase in time, but at time equal to 8.3s the output tend to drop and at time above 8.3s the output rate is increased. Also for equal time range (0 to 10s) and varying area we have different value of output rate, we can also see that increase in area also result to increase in output response from the figure above. The behavior is due to the properties of the crude, some of this properties are the viscosity of the crude oil, the sulphur content of the crude, the presence of salt in the crude is also a major contributing factor for the behavior of the output response as presented in figure 5a,b,c,d.

**CONCLUSIONS**

The main points concluded from the present study are summarized as follows:
1) Sensor area is a major factor that influences the kind of control response to be used in the distillation column.
2) The residence time also influence output rate of the column.
3) The type of control response applied in the distillation unit determines if there will be an increase or decrease in the output rate with respect to time.
4) Step input application in distillate and reflux ratio is one of the best methods in monitoring

REFERENCES


